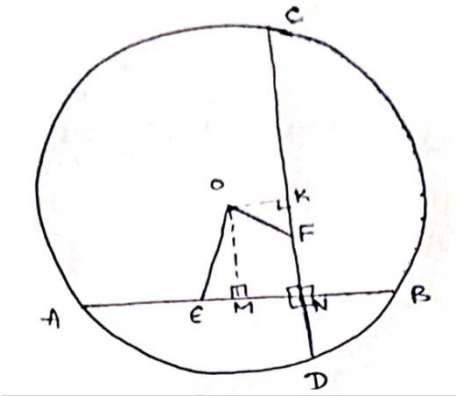
01.01.2024 Cash Award Math Rider : III Prize Winner Ms. Naga Bhavya Sree's Solution

Given:



NB=NE; ND =NF

Let OM, OK are perpendicular bisectors of AB, CD respectively.

$\Longrightarrow AM = MB$

$\Longrightarrow$ NM+NB=AM

NE-EM+NE =AE+EM [as NB=NE]

= 2(NE-EM) = AE ----------(1)

$\Longrightarrow$ KC = KD $\Longrightarrow FC - KF = NF + KF + ND$

2(NF+KF) = FC (as ND = NF) ---------- (2)

In circle ACBD,

$\Longrightarrow AN.NB = ND.NC$

NE (AE+NE) = NF (NF+FC) (as NB= NE ; ND = NF)

From (1) & (2)

NE (3NE-2EM) = NF (3NF+2KF)

$\Longrightarrow 3NE^2 - 3NF^2 = 2EM.NE + 2NF.KF$ ---------- (3)

from question;

$(OE)^2 - (OF)^2$

$= (EM)^2 + (OM)^2 - (OK)^2 - (FK)^2$ (OMNK is a rectangle)

$= (EM)^2 + (NF + FR)^2 - (NE - EM)^2 - (FK)^2$

$= (NF)^2 - (NE)^2 + 2NF.FK + 2NE.EM$

From (3)

$=NF^2 - NE^2 + 3(NE^2 - NF^2)$

$\Longrightarrow OE^2 - OF^2 = 2\,(NE^2 - NF^2)$ ---------- Hence Proved

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